PURE
MATHEMATICS
Paper 1
July/August 2024
3hours

KAMSSA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3hours

Instructions to candidates:

- *Answer **All** the **eight** questions in section **A** and **five** questions from section **B**.*
- *Any additional question (s) answered will not be marked.*
- *All working **must be** shown clearly.*
- *Begin each answer on a fresh page.*
- *Graph paper is provided.*

Silent non-programmeble, scientific calculators and mathematical tables with atleast of formaulae may be used.

- *State the degree of accuracy at the end of each answer given. If a calculator or a mathematical table is used, indicate **Cal** for calculator or **Tab** for mathematical tables.*

Turn over

SECTION A

Answer all the questions in this section

✓1) Given $x + 2x^3 + 4x^5 + 8x^7 + \cdots = \frac{3}{7}$

i) Find the value of x. (03 marks)

ii) Find the 20^{th} term. (02 marks)

✓ 2) The directional vectors $\boldsymbol{a} = 3\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{b} = -2\boldsymbol{i} + \boldsymbol{j} + p\boldsymbol{k}$ and $\boldsymbol{c} = 9\boldsymbol{i} + 9\boldsymbol{j}$ are such that $\boldsymbol{a}$ is perpendicular to $\boldsymbol{b}$. Find the

i) Value of the scalar $\boldsymbol{p}$. (02 marks)

ii) Angle between $\boldsymbol{b}$ and $\boldsymbol{c}$. (03 marks)

3) If $y = 2\sin\theta$ and $x = \cos 2\theta$, show that $\frac{d^2y}{dx^2} = \frac{-1}{y^3}$ (05 marks)

✓ 4) Given $A(7,3)$ and $B(-5,0)$, the line through points A and B meets the line $3x + 5y = 19$ at point C. What are the coordinates of C. (05 marks)

✓ 5) The roots of the equation $x^2 + 6x + 3 = 0$ are α and β. Find the equation whose roots are $\frac{\alpha+\beta}{\alpha}$ *and* $\frac{\alpha+\beta}{\beta}$ (05 marks)

6) Show that the area bounded by the curve $y = 10x - x^2$ and the line $y = 4x$ is 36 square units. (05 marks)

✓ 7) Solve $\tan^{-1} x + \tan^{-1}(x-1) = \tan^{-1} 3$ (05 marks)

8) Find the value of x and y such that

$3\log_8 xy = 4\log_2 x$ *and* $\log_2 y = 1 + \log_2 x$ (05 marks)

1 2 3

$y = 4x$
$y = 4 \times 1$ (1,4) (2,8) (3,12)

SECTION B

Answer any five questions from this section. ***All questions carry equal marks***

9 a) The binomial expansion of $(1 + kx)^n$ is $1 - 6x + 30x^2 + \cdots$

find the value of k and n $(1+4x)^{3/2}$ 1+ $(1-4x)^{3}$ (05 marks)

b) Expand $\sqrt{1+8x}$ up to the term in x^3. Using $x = \frac{1}{100}$, show that $\sqrt{3}$=1.73205

$(1+x)^6 = 1-$ 1+ (07 marks)

10 (a) Integrate $e^x \sin 2x$ with respect to $\boldsymbol{x}$. $1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!}$ (06 marks)

$1 + nx$

(b) Using t =tanx or otherwise, evaluate $\int_0^{\frac{\pi}{3}} \frac{1}{1+8\cos^2 x} dx$. $1 + \frac{8x^2}{100}$ (06 marks)

$1 + \frac{2}{25}$ Sin A + Sin(A+B)

Turnover

(11) The lines $x = \frac{y-1}{2} = \frac{z-21}{-3}$ and $\boldsymbol{r} = (12+3\lambda)\boldsymbol{i} + (7-3\lambda)\boldsymbol{j} + (1-\lambda)\boldsymbol{k}$ lie in the same plane.

(a) Show that the lines intersect and calculate the angle between them (06 marks)

(b) Find the equation of the plane where both lines lie. (06 marks)

12)(a) Show that. $\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$.

Hence solve $\frac{2\tan\theta}{1+\tan^2\theta} = 3\cos 2\theta$ for $0 \le \theta \le \pi$. (06 marks)

b) (i) Express $7\cos x - 24\sin x$ in the form $Rcos(x+a)$. (04 marks)

(ii) Write down the maximum and minimum value of the function

$f(x) = 12 + 7\cos x - 24\sin x$. (02 marks)

13)(a) Given $z = -1 + i\sqrt{3}$, find the value of the real number p such that

$$Arg(z^2 + Pz) = \frac{5\pi}{6}$$ (05 marks)

(b) if $\frac{|w-1|}{|w+2|} < 2$ where $w = x + iy$, represent the locus of w on the argand diagram.

(14) Sketch the curve of the equation $y = \frac{x^2+3}{x-1}$ (12 marks)

(15) An ellipse has a Cartesian equation

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

The general point $P(4\cos\theta, 2\sin\theta)$ lies on the ellipse.

(a) Show that the equation of the normal to the ellipse at P is

$2x\sin\theta - y\cos\theta = 6\sin\theta\cos\theta$ (06 marks)

b) The normal to the ellipse at P meets the x axis at the point Q and O is the origin. Show clearly that as θ varies, the maximum area of the triangle OPQ is $4\frac{1}{2}$. (06 marks)

16) (a) solve $xy\frac{dy}{dx} = x^2 + y^2$, using $y = ux$. (04 marks)

(b) A liquid is pouring into a container at a constant rate of $20cm^3s^{-1}$ and is leaking out at a rate proportional to the volume, $v\ cm^3$ of the liquid already in the container.

The volume $v\ cm^3$ of the liquid in the container at any time t has a differential equation

$$\frac{dv}{dt} = 20 - kv$$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$